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VERSITÄT

SignVectors: a SageMath Package for Oriented Matroids

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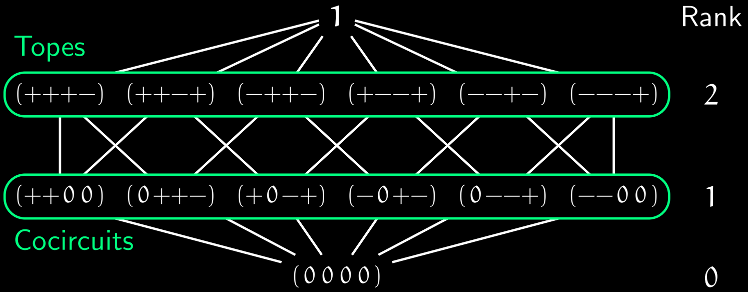
github.com/MarcusAichmayr/sign_vectors

Crash Course on Oriented Matroids

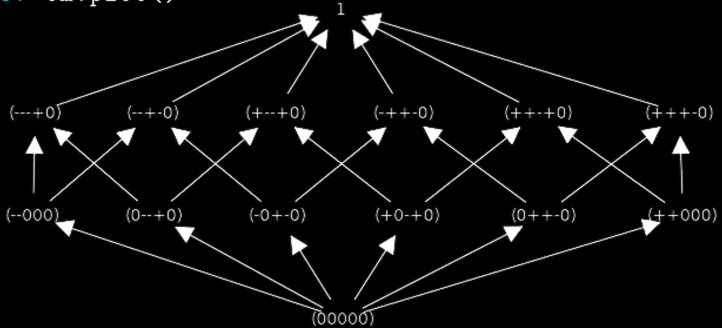
- Sign vector: Element in $\{-, 0, +\}^n$ $\qquad$ $\text{sign}\begin{pmatrix}-1\\0\\2\end{pmatrix} = (-0+)$
- Oriented matroid: Sign vectors of real subspace
  - $\rightarrow$ Fulfills covector axioms
- Partial order on $\{-, 0, +\}^n$ by $0 \preceq -, +$
  - ◦ Add top element $\rightarrow$ Big face lattice



Package SignVectors

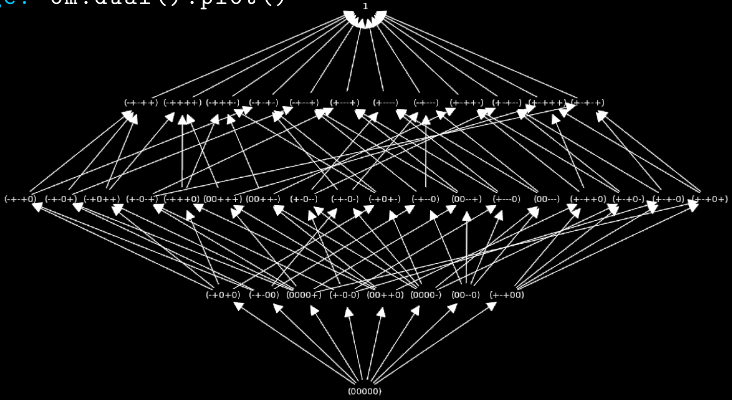
TODO explain how we get from a subspace to a matrix

```
sage: from sign_vectors import *
sage: P = matrix([[1, 2, 0, 0, 0], [0, 1, 2, -3, 0]])
sage: om = OrientedMatroid(P) # instant

sage: om.cocircuits()
{(-0+-0), (0++-0), (--000), (++000), (0--+0), (+0-+0)}
sage: om.topes()
{(---+0), (+--+0), (--+-0), (-++-0), (++-+0), (+++-0)}
sage: om.plot()
```



Duality

```
sage: om.circuits() # cocircuits of dual
{(0000+), (0000-), (00++0), (-+0+0), (00--0), (+-0-0),
    (-+-00), (+--+00)}

sage: om.dual()
Oriented matroid of rank 3 with elements of size 5.
sage: om.dual().plot()
```



Constructions

- Chirotope: Sign pattern of oriented matroid

```
sage: om.chirotope()
[+, +, -, 0, +, -, 0, 0, 0, 0]
```

→ Create from chirotope

```
sage: OrientedMatroid.from_chirotope("++-0+-0000", 2, 5)
Oriented matroid of rank 2 with elements of size 5.
```

→ Create from

- Circuits
- Cocircuits
- Topes
- ...

```
sage: om.matroid() # underlying matroid
Matroid of rank 2 on 5 elements with 5 bases
```

Other Software

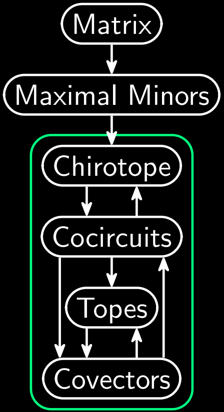
- TOPCOM
  - C++
  - Commandline
  - For integers
- OrientedMatroids
  - Also SageMath
  - Beta
  - Verifies axioms

Table: Runtimes (in s) for computing circuits from matrices

Ring	Size	Chirotope	ours	TOPCOM	OrientedMatroids
$\mathbb{Z}$	5×10	$3 \cdot 10^2$	0.013	0.017	2.4
	5×20	$2 \cdot 10^4$	1.6	1.2	> 3600
	10×20	$2 \cdot 10^5$	11	42	–
	13×26	$1 \cdot 10^7$	720	5000	–
$\overline{\mathbb{Q}} \cap \mathbb{R}$	5×10	$3 \cdot 10^2$	0.95	–	2.0
	5×15	$3 \cdot 10^3$	14	–	crash

Summary

- Implementation
  - Object-oriented
  - Lazy computation
  - Caching
  - Bit sets
- Documentation
  - Fully documented
  - Many examples
  - Automatically generated
- Automated doctests



Oriented Matroid

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